

**B.Tech. Degree I & II Semester Examination in
Marine Engineering May 2014**

MRE 1102 ENGINEERING MATHEMATICS-II

Time : 3 Hours

Maximum Marks : 100

- I. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 4 & 5 & 8 \\ 3 & 1 & 2 & 3 \end{bmatrix}$ (6)
- (b) Test the consistency of the system of equations. (6)
- $$\begin{aligned} x - y + 2z &= 2 \\ 2x + y + 4z &= 7 \\ 4x - y + z &= 4 \end{aligned}$$
- and hence solve them (if possible).
- (c) Find the eigen values and eigen vectors of the matrix. (8)
- $$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$
- OR**
- II. (a) Using De Moivre's Theorem prove that (6)
- $$(a+ib)^{\frac{m}{n}} + (a-ib)^{\frac{m}{n}} = 2(a^2+b^2)^{\frac{m}{2n}} \cos \left[\frac{m}{n} \tan^{-1} \frac{b}{a} \right]$$
- (b) Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$ where C is the circle (6)
- (i) $|z|=1$
- (ii) $|z+1-i|=2$
- (iii) $|z+1+i|=2$.
- (c) Find the Laurent's expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < z+1 < 3$ (8)
- III. (a) Solve $\frac{dy}{dx} - y \tan x = \frac{\sin x \cdot \cos^2 x}{y^2}$ (6)
- (b) Solve $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$ (7)
- (c) By method of variation of parameters solve the equation $\frac{dy^2}{dx^2} + y = \sec x$ (7)

OR

(P.T.O.)

IV. (a) Solve $(D^2 + 4D + 5)y = e^x + x^3 + \cos 2x$ (8)

(b) Solve $(D^2 + 3D + 2)y = e^{2x} \cdot \sin x$ (6)

(c) Solve the simultaneous equations (6)

$$\frac{dx}{dt} + 2x + 3y = 0$$

$$\frac{dy}{dt} + 3x + 2y = 2e^{2t}$$

V. (a) Find the Fourier series for $|x|$ in $(-\pi, \pi]$ and deduce that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ (10)

(b) If $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & x > \frac{\pi}{2} \end{cases}$ (10)

Expand $f(x)$ as a sine series in $(0, \pi)$

OR

VI. (a) Show that $\frac{\beta(p, q+1)}{q} = \frac{\beta(p+1, q)}{p} = \frac{\beta(p, q)}{p+q}$ (6)

(b) Evaluate $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta \cdot \int_0^{\pi/2} \frac{1}{\sqrt{\sin \theta}} d\theta$ (6)

(c) Prove the following

(i) $\int_0^1 \frac{1}{\sqrt{x}} dx = \sqrt{\pi}$ (4)

(ii) $\int_0^1 \sqrt{n+1-x} dx = n \int_0^1 \sqrt{x} dx$ (4)

VII. (a) Find the Laplace transforms of

(i) $L\left\{\frac{\sin at}{t}\right\}$ (5)

(ii) $L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$ (5)

(b) Find inverse Laplace transforms of

(i) $L^{-1}\left\{\frac{S}{(S^2 + a^2)^2}\right\}$ (5)

(ii) $L^{-1}\left\{\frac{1}{(s+1)(s^2 + 2s + 2)}\right\}$ (5)

OR

VIII. (a) Solve $y'' - 3y' + 2y = 4e^{2t}$; given $y(0) = -3$, $y'(0) = 5$ (10)

(b) Solve the simultaneous equations (10)

$$x'' + y' + 3x = 15e^{-t}$$

$$y'' - 4x' + 3y = 15 \sin 2t$$

Subject to

$$x(0) = 35, \quad x'(0) = -48$$

$$y(0) = 27, \quad y'(0) = -55$$

(Contd...3.)

- IX. (a) First bag contains 5 red balls and 3 black balls, second bag contains 4 red balls and 7 black balls. One ball is drawn from each bag. Find the probability that (6)

- (i) both are red
(ii) one is red and one is black

- (b) The probability density function of a variate X is (8)

X	0	1	2	3	4	5	6
p(X)	k	3k	5k	7k	9k	11k	13k

- (i) Find $P(x < 4)$, $P(x \geq 5)$, $P(3 < X \leq 6)$

- (ii) What will be the minimum value of k so that $P(x \leq 2) > 0.3$

- (c) A variate X has the probability distribution (6)

$$x : -3 \quad 6 \quad 9$$

$$P(X = x) : \frac{1}{6} \quad \frac{1}{2} \quad \frac{1}{3}$$

Find $E(x)$ and $E(x^2)$.

Hence evaluate $E\{(2x+1)^2\}$.

OR

- X. (a) The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that (5)

- (i) exactly two will be defective
(ii) At least two will be defective.

- (b) A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiased die? (5)

- (c) Equations of two regression lines are $4x+3y+7=0$ and $3x+4y+8=0$ (10)

Find (i) mean of x, mean of y

(ii) Regression coefficients b_{yx} and b_{xy}

(iii) Correlation coefficient between x and y.
